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ABSTRACT

Data banks with items calibrated under an item response model can be used for the construction of tests. Mathematical programming models like the Maximin Model are formulated for computerized item selection from a bank. In this paper, mathematical programming models based on the Maximin Model are proposed for the construction of weakly parallel tests. Numerical experiments were conducted to obtain an impression of the practicality of the approach using an item bank of 600 items from college placement mathematics examinations (520 items were from 13 previously administered American College Testing Assessment Program tests, and 80 were from the Collegiate Mathematics Placement Program). Six tests were constructed, and their test information functions were computed. The results demonstrate that tests constructed with the proposed models were near-optimal with respect to the Maximin criteria and were approximately weakly parallel. Five tables present information about the constructed tests. Four graphs illustrate the information functions. (Author/SLD)

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The Construction of Weakly Parallel Tests by Mathematical Programming

Research Report 90-6

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The Construction of Weakly Parallel Tests

by Mathematical Programming

Jos J. Adema

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Abstract

Databanks with items calibrated under an item response model can be used for the construction of tests. Mathematical programming models like the Maximin Model are formulated for computerized item selection from a bank. In this paper mathematical programming models based on the Maximin Model are proposed for the construction of weakly parallel tests. Numerical experiments have shown that tests constructed with the proposed models are near-optimal with respect to the maximin criterion and are approximately weakly parallel.

Keywords: Item Banking; Test Construction; Mathematical Programming; Weakly Parallel Tests.

The Construction of Weakly Parallel Tests by Mathematical Programming

A new development in item response theory is the construction of tests by mathematical programming models (e.g., Adema & van der Linden, 1989; Baker, Cohen & Barmish, 1988; Boekkooi-Timminga, 1989; Theunissen, 1985; van der Linden & Boekkooi-Timminga, 1989). These models are used to select items from a bank calibrated under an item response model, such that the constructed test is in some sense (approximately) optimal. The models also take into account that all the demands of the test constructor with repect to, for instance, test composition and administration time are satisfied. Boekkooi-Timminga (1987, 1990) has proposed a number of mathematical programming models for the construction of weakly parallel tests. Samejima (1977) defines two weakly parallel test forms as "a pair of tests which measure the same ability and whose test information functions are identical".

Two main approaches can be distinguished in the construction of weakly parallel tests by mathematical programming: (1) Sequential and (2) Simultaneous test construction (Boekkooi-Timminga, 1990). In the sequential case the tests are constructed one after the other. Each time the best items are selected, which implies that the psychometric quality of the tests is likely to decrease in the order of construction. For the Rasch model the decrement in quality can be small, but for the 3-parameter model the decrement in general is large. So the disadvantage of this approach is that the constructed tests are not always fully parallel. In the simultaneous case all the tests are constructed at the same time. Simultaneous constructed tests are approximately weakly parallel (exact weakly parallel tests do not exist in practice because test information functions are never equal). The drawback of the simultaneous approach are the large amount of computer time and storage required.

In this paper mathematical models are proposed that allow weakly parallel tests to be constructed sequentially. The advantage of the new approach is that the constructed tests are approximately weakly parallel and that it is not as time and storage consuming as simultaneous test construction.

In most of the test construction models available the test information function plays an important role. The test information function for an unbiased estimator of ability is defined as the reciprocal of the (asymptotic) sampling variance of the estimator (Lord, 1980) which makes it a measure of the quality of a test. Also the feature that the test information function can be computed by addition of the item information function is very useful:

$$I(\theta) = \sum_{i=1}^{n} I_i(\theta),$$

where θ is the ability parameter, n the number of items in the test, and $I_i(\theta)$ the information function of item i.

The mathematical programming models as formulated in this paper are based on the Maximin Model (van der Linden & Boekkooi-Timminga, 1989), which is applicable for the construction of one test at a time. A brief review of the Maximin Model is given here. Associated with each item is a decision variable x_i such that

$$x_i = \begin{cases} 0 \text{ item } i \text{ not in the test} \\ 1 \text{ item } i \text{ in the test} \end{cases} \qquad i = 1, \ldots, I,$$

where I is the number of items in the item bank. The information function of the test to be constructed is only considered at a number of ability levels θ_k, $k = 1, \ldots, K$. The test constructor can choose both the number and spacing of these levels. Let $I_i(\theta_k)$ denote the information function value of item i at ability level θ_k. The test constructor has to specify the relative shape of the target test information function by choosing constants r_k, $k = 1, \ldots, K$. Let y be decision variable such that $(r_1 y, \ldots, r_K y)$ is a series of lower bounds to the test information function at the ability levels θ_k. If n is the prescribed number of items in the test then the Maximin Model is formulated as follows:

(1) Maximize y,

subject to

(2) $\sum_{i=1}^{I} I_i(\theta_k)x_i - r_k y \geq 0,$ $\qquad k = 1,\ldots, K,$

(3) $\sum_{i=1}^{I} x_i = r,$

(4) $\sum_{i=1}^{I} a_{ij}x_i = b_j,$ $\qquad j = 1,\ldots, J,$

(5) $x_i \in \{0,1\},$ $\qquad i = 1,\ldots, I,$

(6) $y \geq 0.$

Constraints (4) are a general provision for practical constraints such as constraints on the administration time, test composition etc. By maximizing y in the objective function (1) the lower bounds $(r_1y,\ldots, r_Ky)$ are forced to be as high as possible (max-part). By imposing the constraints (2) $(r_1y,\ldots, r_Ky)$ is a series of lower bounds to the test information function (min-part). The number of items in the test is controlled by constraint (3).

The Maximin Model is known in the operations research literature (e.g. Wagner, 1975; Hartley, 1985) as a mixed integer linear programming (MILP) model, because the objective function and constraints are linear in the decision variables and there are continuous (y) as well as integer (x_i) variables in the model. A well-known method for solving MILP models is the branch-and-bound method (Land & Doig, 1960). In its standard form this method is time-consuming. Adema, Boekkooi-Timminga, and van der Linden (in press) have proposed a heuristic based on the branch-and-bound method for solving test construction models, that solves the Maximum Model for large item banks in favorable time. The heuristic will be used in forthcoming numerical experiments.

In the next section a mathematical programming model for simultaneous test construction is formulated. The model is not recommended for practical application, but will be used to evaluate the quality of forthcoming models in the discussion section. Then mathematical programming models are presented to construct tests sequentially. Next, these models are used in numerical experiments to get an impression about the practicability (CPU-time and accuracy) of the approach presented in this paper. The results of the experiments are evaluated in the discussion section.

A Simultaneous Test Construction Model

In this section a mixed integer linear programming model is presented for constructing tests simultaneously.

Suppose T tests have to be constructed and define the decision variables x_{it} as:

$$x_{it} = \begin{cases} 0 & \text{item } i \text{ not in test } t \\ 1 & \text{item } i \text{ in test } t. \end{cases} \qquad \begin{array}{l} i = 1, .., I; \\ t = 1, .., T \end{array}$$

The Maximin Model can easily be extended to the case of simultaneous test construction by taking $(r_1y, \ldots, r_Ky)$ as a series of lower bounds for the test information function of all T tests:

(7) Maximize y,

subject to

(8) $\sum_{i=1}^{I} I_i(\theta_k)x_{it} - r_ky \geq 0,$ $\quad k = 1, \ldots, K; \; t = 1, \ldots, T,$

(9) $\sum_{i=1}^{I} x_{it} = n,$ $\quad t = 1, \ldots, T,$

(10) $\sum_{i=1}^{I} a_{ij}x_{it} = b_j,$ $\quad t = 1, \ldots, T; \; j = 1, \ldots, J,$

(11) $\sum_{t=1}^{T} x_{it} \leq 1,$ $\quad i = 1, \ldots, I,$

(12) $x_{it} \in \{0, 1\},$ $\quad i = 1, \ldots, I; \; t = 1, \ldots, T,$

(13) $y \geq 0.$

Constraints (8) imply that $(r_1y, \ldots, r_Ky)$ is a series of lower bounds for all T test information functions. The number of items in the tests are equal to n by constraints (9). Constraints (10) are a general notation for possible practical constraints. To preclude that items are selected for more than one test constraints (11) are imposed.

The main disadvantage of model (7)-(13) is that the number of variables and constraints increases rapidly if the number of tests increases. Thus solving model (7)-(13) can be time consuming and cost a lot of computer storage. In the next section MILP models with results comparable to model (7)-(13) are formulated. The new MILP models are, however, less time and computer storage consuming than model (7)-(13). The results of model (7)-(13) will not always yield (approximately) weakly parallel tests, because the test information functions are only bounded from below. However, model (7)-(13) can be generalized such that bounds from above are included (Boekkooi-Timminga, 1990). As model (7)-(13) is a generalized version of all other simultaneous models including bounds from above, its objective function will be highest. This objective function value will be used for evaluating the quality of the forthcoming models in the discussion section.

New Models for the Construction of Weakly Parallel Tests

In this section MILP models for constructing weakly parallel tests sequentially are presented. These models contain extra constraints that, at previous stages, allow for tests to be constructed at later stages.

Next, the MILP model for constructing the first test is formulated. The basic idea is as follows: Suppose T tests have to be constructed. The model for the first test is used for constructing two tests, namely, the first test and a dummy test that is T-1 times the size of this first test. In the latter test the decision variables are allowed to take non-integer values. The large test actually represents the T-1 tests that have to be constructed after the first test. The decision variables corresponding to the first test are

$$x_i = \begin{cases} 0 & \text{item } i \text{ not in the first test} \\ 1 & \text{item } i \text{ in the first test} \end{cases}$$

and for the other test the decision variables z_i are introduced, where z_i is the fraction of item i in the large test. The model is as follows:

(14) Maximize y,

subject to

(15) $\sum_{i=1}^{I} I_i(\theta_k)x_i - r_k y \geq 0, \qquad k = 1,\ldots, K,$

(16) $\sum_{i=1}^{I} x_i = n,$

$$(17)\qquad \sum_{i=1}^{I} a_{ij}x_i = b_j, \qquad j = 1,\ldots, J,$$

$$(18)\qquad \sum_{i=1}^{I} I_i(\theta_k)z_i - (T-1)r_k y \geq 0, \qquad k = 1,\ldots, K,$$

$$(19)\qquad \sum_{i=1}^{I} z_i = (T-1)n,$$

$$(20)\qquad \sum_{i=1}^{I} a_{ij}z_i = (T-1)b_j, \qquad j = 1,\ldots, J,$$

$$(21)\qquad x_i + z_i \leq 1, \qquad i = 1,\ldots, I,$$

$$(22)\qquad x_i \in \{0,1\}, \qquad i = 1,\ldots, I,$$

$$(23)\qquad z_i \geq 0, \qquad i = 1,\ldots, I,$$

$$(24)\qquad y \geq 0.$$

The decision variables x_i in model (14)-(24) denote which items are selected for the first test. The objective function (14) and constraints (15)-(17), (22) and (24) give the basic Maximin Model. By inclusion of constraints (18)-(21), and (23) the best items are prevented from being selected only in the first test. Constraints (21) imply $z_i \leq 1$ and prevent items from being selected for the first and the dummy test.

For the construction of the second test model, (14)-(24) is applied again with T-2 instead of T-1 and with the items selected for the first test deleted from the bank. The decision variables now denote which items will be selected for the second test.

In an analogous way the remaining tests are constructed. If in constraints (18)-(20) T-1 is replaced by T-t, the complete test construction procedure is as given by Algorithm A:

Algorithm A

Step 1: t := 1;

Step 2: Solve model (14)-(24); The selected items from the item bank represent test t.

Step 3: If t = T then STOP else delete the selected items from the item bank, t := t+1 and go to Step 2.

The tests constructed according to the above approach are not necessarily weakly parallel tests, because the test information functions are not bounded from above. The problem occurs when for one or more θ_k's the chosen r_k's are such low that the corresponding constraints in (15) and (18) in the relaxed model (14)-(24) are not active, i.e., that for the optimal solution to model (14)-(24) with $0 \leq x_i \leq 1$ instead of $x_i \in \{0,1\}$ the test information function value is above $r_k y$ for some θ_k's. To resolve this problem Algorithm B is proposed. In this algorithm the values of r_k that are too low are increased such that the constraints in (15) and (18) are more restrictive:

Algorithm B

Step 1: $t := 1$;

Step 2: Solve the relaxed model (14)-(24);

Step 3: Adjust r_k, $k = 1, \ldots, K$:

$$r_k = \left(\sum_{i=1}^{I} I_i(\theta_k) x_i + \sum_{i=1}^{I} I_i(\theta_k) z_i \right) (Ty)^{-1},$$

where x_i $(i = 1, \ldots, I)$, z_i $(i = 1, \ldots, I)$, and y are computed in Step 2;

Step 4: Solve model (14)-(24). The selected items from the item bank represent test t.

Step 5: If $t = T$ then STOP else delete the selected items from the item bank, $t := t+1$ and go to Step 4.

In Step 3 the value of r_k is adjusted only at ability levels θ_k with $\Sigma I_i(\theta_k) x_i > r_k y$ and/or $\Sigma I_i(\theta_k) z_i > (T-1) r_k y$ such that large differences between test information functions are less likely, because of the more restrictive constraints (15) and (18).

To make the approach even better - but also more time consuming because the model will be more restricted - it is possible to put an upper bound on

$$\sum_{i=1}^{I} I_i(\theta_k)x_i - r_k y.$$

How to impose this upper bound is explained next. Let e_k, $k = 1,\ldots, K$ be decision variables that are equal to the test information function value at θ_k minus $r_k y$. Then it is required that:

$$(25) \qquad \sum_{i=1}^{I} I_i(\theta_k)x_i - r_k y - e_k = 0, \qquad k = 1,\ldots, K.$$

If constraints (15) are replaced by constraints (25), the test information function can be bounded from above by imposing upper bounds on the variables e_k:

$$(26) \qquad 0 \le e_k \le E_k, \qquad k = 1,\ldots, K,$$

where E_k is a prespecified upper bound that guarantees a required precision.

Numerical Experience

It is hard to solve mixed integer linear programming models in general. Therefore, the heuristic as proposed by Adema (1988) will be used in this section. The heuristic is based on the branch-and-bound (BAB) method (Land & Doig, 1960). A full explanation of the heuristic is beyond the scope of this paper. However, two important parameters (H_1 and H_2) are described here. The branch-and-bound method as well as the heuristic start with solving the relaxed model ($0 \le x_i \le 1$ instead of $x_i \in \{0,1\}$) by standard linear programming. In the heuristic the model is, then, reduced by fixing variables according to the following rules ($H_1 < 1$ is prespecified):

(1) Fix x_i to 0, if in the relaxed solution $x_i = 0$ and $(1 - H_1) z_{LP} < d_i$, where z_{LP} is the objective function value for the optimal solution to the relaxed model and d_i the reduced cost of variable x_i (See e.g. Murtagh, 1981, p.25);

(2) Fix x_i to 1, if in the relaxed solution $x_i = 1$ and $(1 - H_1)\ z_{LP} < -d_i$.

Anotner feature of the heuristic is that that the difference between z_{LP} and the objective function value of the optimal 0-1 solution is required to be smaller than $(1 - H_2)*100\%$ of z_{LP}, where $H_2 < 1$ is also prespecified. The above modifications speed up the search process after the relaxed model is solved. A last modification is to stop as soon as an 0-1 solution - not necessarily the optimal 0-1 solution - has been found. The objective function value of the 0-1 solution found will be between $H_2 * z_{LP}$ and z_{LP}. So if H_2 is close to 1 the heuristic will give a nearly optimal 0-1 solution.

MPSX/370 V2 is an IBM licensed program for handling linear and mixed integer linear programming problems (IBM MPSX/370 V2 Program Reference Manual, 1988). It provides the user with algoritmic tools which enable him/her to build his/her own heuristics and algorithms by writing a control program in ECL, a computer programming language based on

PL\1. The heuristic together with Algorithms A and B were implemented in ECL programs. The CPU-times in the forthcoming tables are the execution times of the ECL programs on an IBM9370 computer.

A short description of the item bank used in the experiments is given below. Ackerman (1989) gives a more detailed description of the item bank. It consisted of 600 items; 520 items were from 13 previously administered ACT Assessment Program (AAP) tests and 80 were from the Collegiate Mathematics Placement Program (CMMP). The items were calibrated under the 3-parameter logistic model (Birnbaum, 1968). Thus, the probability of an examinee with ability level θ to answer an item i correctly was given by:

$$P_i(\theta) = c_i + (1-c_i)\ (1 + \exp(-Da_i(\theta-b_i))^{-1},$$

where D is a constant equal to 1.7 and a_i, b_i, and c_i are the discrimination, difficulty, and guessing parameter of item i. The information function is expressed by:

$$I_i(\theta) = \frac{D^2 a_i{}^2 (1-c_i)}{(c_i+\exp(Da_i(\theta-b_i)))\ (1+\exp(-Da_i(\theta-b_i)))^2}.$$

The bank was partitioned in six content areas:

(1) Arithmetic and Algebraic Operations (AAO);

(2) Arithmetic and Algebraic Reasoning (AAR);

(3) Geometry (G);

(4) Intermediate Algebra (IA);

(5) Number and Numeration Concepts (NNS);

(6) Advanced Topics (AT).

From the bank items were selected to create weakly parallel tests with 40 items (4 AAO items, 14 AAR items, 8 G items, 8 IA items, 4 NNS items, and 2 AT items). The six ability levels and relative information function values in the test construction models (θ_k, r_k) were: (-1.6,2.0), (-.8,5.4), (.0,12.1), (.8,21.3), (1.6,10.8), and (2.4,3.1).

Tests were constructed using model (14)-(24) (Algorithm A) according to the above test specifications. In Table 1 the differences between the information function values of the tests giving the most and least information at θ_k, $k = 1, \ldots, 6$ are given for T ranging from 2 to 6. The y_{min} value in the table is the lowest objective function value found for the T constructed tests, i.e., $r_k y_{min}$ is a lower bound for the test information function at θ_k for all T tests.

Insert Table 1 here

For the sake of illustration, the test information functions for T = 6 are shown in Figure 1.

Insert Figure 1 here

The large difference found for $\theta_5 = 1.6$ in Table 1 is caused by the steepness of the information functions at the ability level, as can be seen in Figure 1.

Table 2 is similar to Table 1. However, in this case Algorithm B was used. The adjusted r_k values (r_k') are given in Table 3.

Insert Table 2 and 3 here

Figure 2 depicts the test information functions for $T = 6$.

Insert Figure 2 here

The test giving most information at $\theta_6 = 2.4$ contained an item with item information function value 1.810 at this ability level. This item is responsible for the large differences at θ_6 for $T = 5$ and 6.

In Table 4 the results are displayed for the case of the r_k values adjusted and upper bounds (See constraints (25) and (26)) imposed. The upper bounds E_k, $k = 1, .., K$, were computed after Step 2 of Algorithm B was executed as:

$$E_k = 0.025 z_{LP} r_4.$$

In specifying the upper bounds r_4 was used, because at θ_4 most information was wanted. Other ways of specifying the upper bounds are also feasible and may give good results.

Insert Table 4 here

Figure 3 depicts the test information functions for T = 6.

Insert Figure 3 here

Table 4 shows that imposing upper bounds is very effective in making the tests more weakly parallel.

Discussion

In this section three points are discussed: a) How good are the constructed tests with respect to the Maximin criterion?; b) How weakly parallel are the tests?; c) How much CPU-time is needed to solve the models?

Let y' be the objective function value for the optimal solution of the relaxed model (7)-(13) and y_{0-1} be this value for the optimal 0-1 solution to model (7)-(13). Then y' is an upper bound for y_{0-1} and y_{0-1} is an upper bound for y_{min}, because if the T solutions found by Algorithm A or B are combined they give a 0-1 solution to model (7)-(13), but not necessarily the best one. Hence, y_{min}, the worst of these subsolutions, is certainly smaller than y_{0-1}:

$$y' \geq y_{0-1} \geq y_{min}.$$

On the other hand the objective function value y", for the optimal solution to the relaxed model (14)-(24) for t = 1, is an upper bound for y', because every solution to model (7)-(13) is a solution to model (14)-(24). Implying:

$$y'' \geq y' \geq y_{0-1} \geq y_{min}.$$

The values of y" were computed during the test construction process. They are shown in Table 5 together with the differences between y" and y_{min} in percentages of the first.

Insert Table 5 here

The differences are rather small given the fact that y" is an upper bound and y_{min} is the minimum taken over the T constructed tests.

Ackerman (1989) has constructed 6 parallel tests by a simple heuristic with the item bank used here. He does not apply the Maximin criterion but has a fixed target test information function, therefore comparisons are not possible. However, a comparison with repect to the second question in this section can be made.

Insert Figure 4

Figure 4 is a reproduction of the figure of the 6 information functions in the paper of Ackerman. The results in Figure 1 are not as good as the results by Ackerman. Figure 2 and 3, however, are an improvement on the Ackerman results. Especially when upper bounds (constraints (25) and (26)) are imposed, the tests in this paper are practically strictly weakly parallel.

The automated test construction procedure presented here is time consuming, but the construction of weakly parallel tests by hand would be more time consuming, if not impossible. Also, in practice some of the CPU-time needed for constructing the tests can be regained, because tests can be printed and inspected while other tests are still being constructed. Another point to be made is that the paper concentrates on the formulation of the models, where the heuristic applied was not especially developed for these kinds of models. Hence, probably CPU-time can be gained by the development of more specific heuristics especially designed to solve the proposed models.

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Table 1

Differences between test information functions for weakly parallel tests constructed with model (14)-(24)

		Maximum Difference						
T	y_{min}	θ_1	θ_2	θ_3	θ_4	θ_5	θ_6	CPU time (min)
2	1.476	0.031	0.983	0.877	0.382	0.083	0.523	2.64
3	1.384	0.028	0.594	0.914	1.206	0.269	0.311	6.66
4	1.311	0.059	0.[illegible]87	0.680	0.432	1.587	0.576	10.68
5	1.226	0.138	0.767	0.900	0.819	2.563	0.884	14.34
6	1.161	0.133	1.179	1.443	0.733	3.423	1.269	17.82

Note. $H_1 = 0.999$; $H_2 = 0.975$.

Table 2

<u>Differences between test information functions for weakly parallel tests constructed with model (14)-(24) for adjusted r_k values</u>

		Maximum Difference						
T	y_{min}	θ_1	θ_2	θ_3	θ_4	θ_5	θ_6	CPU time (min)
2	1.481	0.008	0.376	0.240	0.584	0.522	0.306	4.56
3	1.391	0.024	0.440	0.288	0.464	0.172	0.188	8.23
4	1.299	0.152	0.501	0.305	0.673	0.144	0.516	12.19
5	1.207	0.197	0.562	0.849	1.280	1.990	1.079	23.25
6	1.158	0.105	0.503	0.645	0.901	0.670	1.335	33.79

Note. H_1 = 0.999; H_2 = 0.975.

Table 3

Adjusted r_k values

T	r_1'	r_2'	r_3'	r_4'	r_5'	r_6'
2	2.000	5.818	12.100	21.300	10.800	3.100
3	2.000	5.571	12.100	21.300	10.800	3.100
4	2.000	5.514	12.100	21.300	11.119	3.100
5	2.000	5.652	12.433	21.300	11.584	3.100
6	2.000	5.943	13.214	21.300	11.884	3.100

Note. The non-adjusted values are: $r_1 = 2$, $r_2 = 5.4$, $r_3 = 12.1$, $r_4 = 21.3$, $r_5 = 10.8$, and $r_6 = 3.1$.

Table 4

Differences between test information functions for weakly parallel tests constructed with model (14),(16)-(26) for adjusted r_k values

		Maximum Difference						
T	y_{min}	θ_1	θ_2	θ_3	θ_4	θ_5	θ_6	CPU time (min)
2	1.486	0.051	0.784	0.063	0.158	0.139	0.645	5.80
3	1.401	0.024	0.394	0.196	0.323	0.208	0.111	7.54
4	1.292	0.130	0.430	0.518	0.743	0.749	0.542	18.22
5	1.229	0.044	0.207	0.574	0.535	0.496	0.116	26.10
6	1.155	0.209	0.558	0.208	0.579	0.530	0.194	53.10

Note. H_1 = 0.999; H_2 = 0.975. The value of H_1 had to be adjusted to 0.975 for T = 5 and 6 during the construction of the last test.

Table 5

Upper bounds for y_{min}

		y_{min}		
T	y"	Table1	Table2	Table4
2	1.513	2.44%	2.12%	1.78%
3	1.417	2.33%	1.83%	1.13%
4	1.331	1.50%	2.40%	2.93%
5	1.256	2.39%	3.90%	2.15%
6	1.189	2.35%	2.61%	2.86%

Acknowledgements

The author would like to thank ACT and Terry Ackerman for making the item bank available to him. The author also appreciates the help of Jos Rikers and Lorette Bosch in the preparation of the figures.

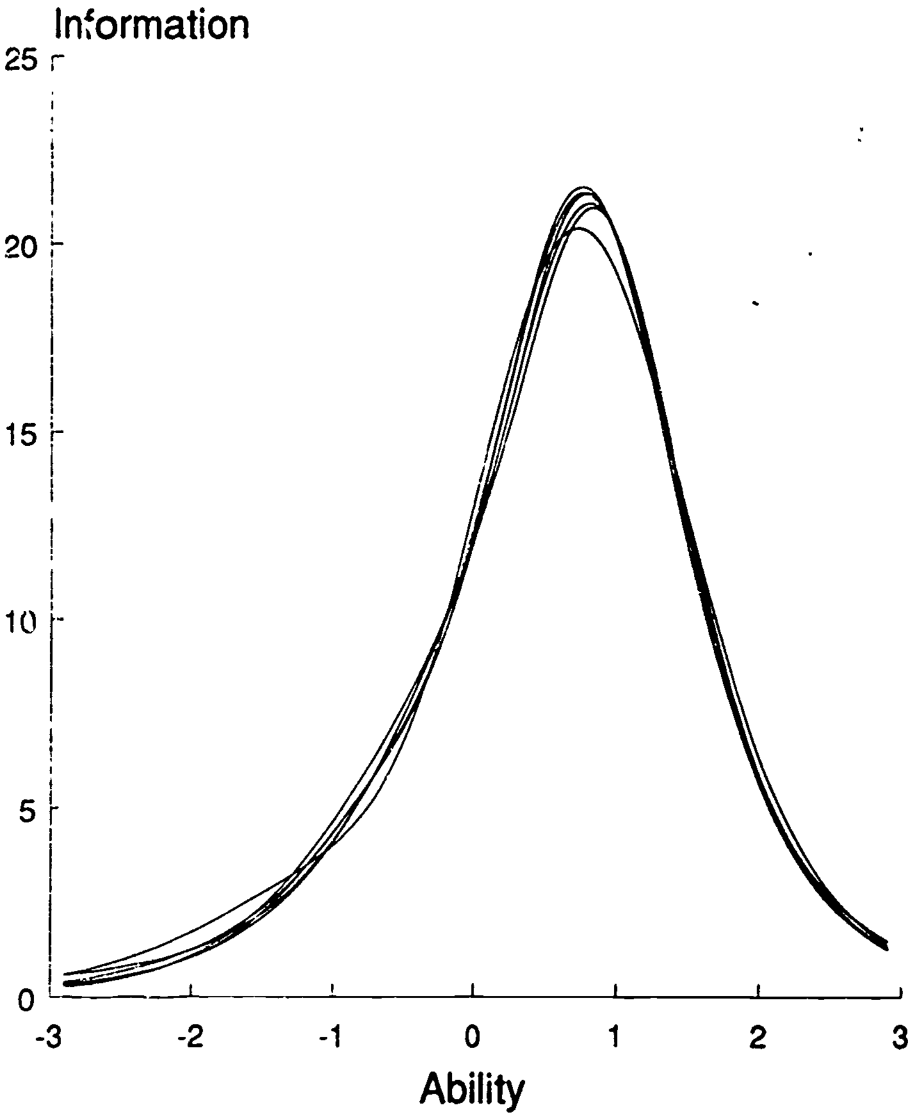
Figure Captions

Figure 1. Information functions of tests constructed with model (14)-(24) for T = 6.

Figure 2. Information functions of tests constructed with model (14)-(24) where the r_k values were modified for T = 6.

Figure 3. Information functions of tests constructed with model (14),(16)-(26) where the r_k values were modified for T = 6.

Figure 4. Information functions of tests constructed by Ackerman. (Note. From "An alternative methodology for creating parallel test forms using the IRT information function" by T.A. Ackerman, 1989. Paper presented at NCME annual meeting. San Francisco. Reprintend by permission.)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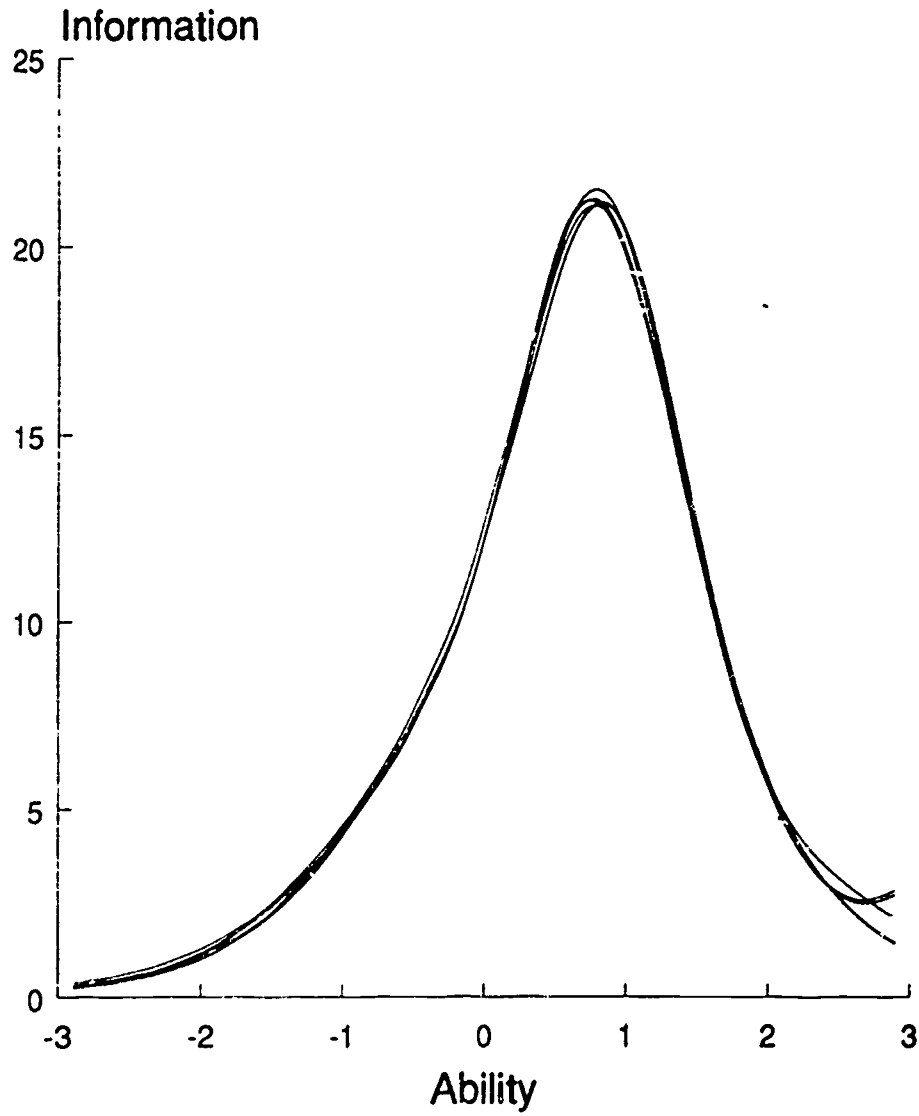
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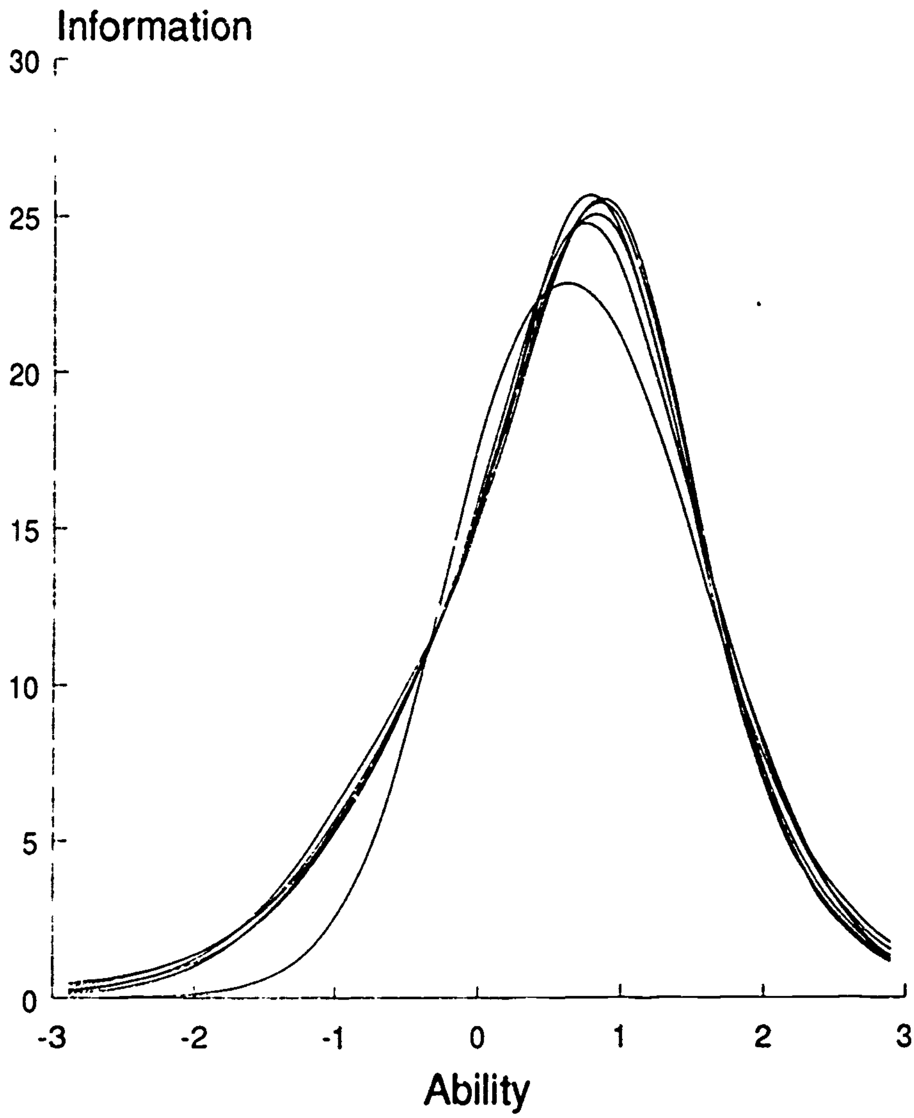
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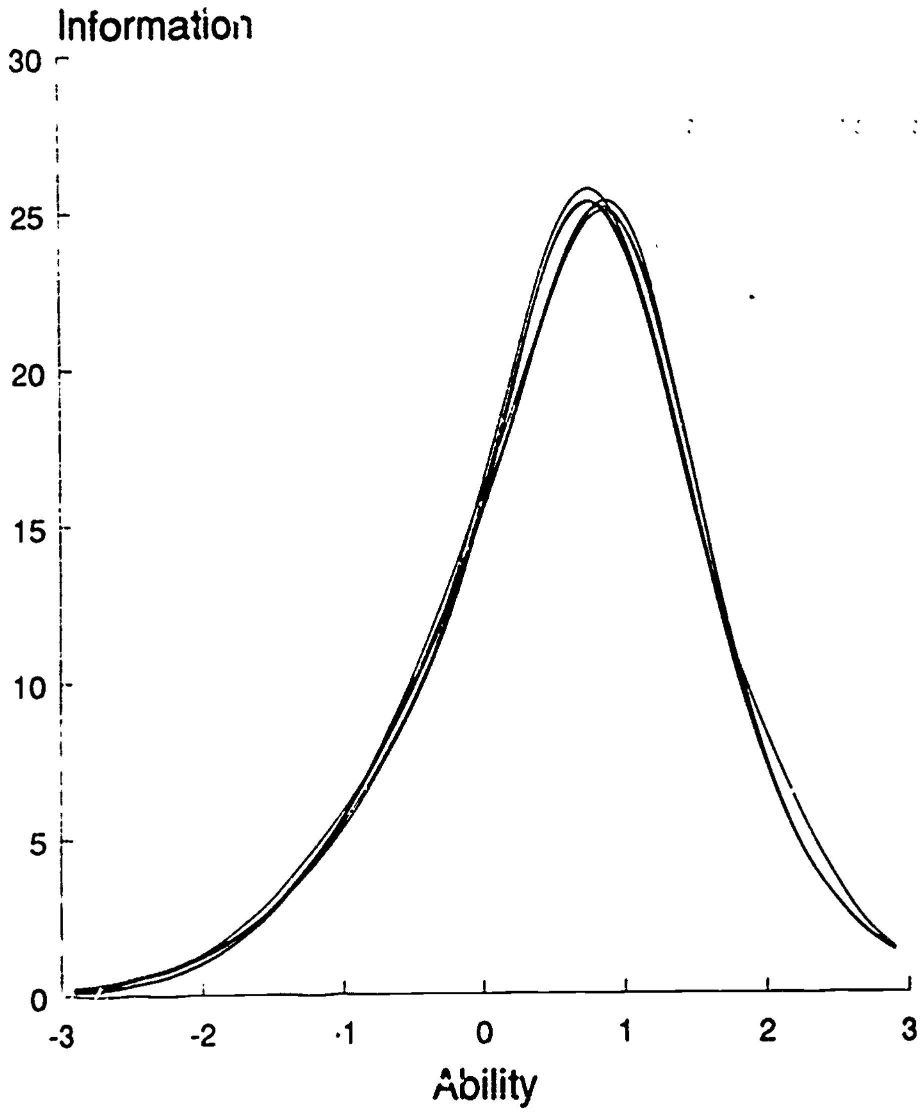
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